Service Life Analysis of Rocket Motors with Internal Gas Generation

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Nomenclature

a_T	= temperature-dependent shift function
D	= gas diffusivity, cm^2/day
d	= day of the year = 1 for Jan. 1
$f(x)$	= distribution function for critical pressure
h	= hour of the day = 16 for 4 p.m.
P	= internal gas pressure in grain, Pa
$\dot{P}$	= time rate of change of internal pressure, Pa/day
P_c	= critical pressure, Pa
P_i	= induced pressure at the end of the ith cycle
$(p_f)_i$	= probability of motor failure during ith cycle
$(P_f)_N$	= probability of motor failure for N cycles
Q	= gas generation rate, cc/g/day
S	= gas solubility, cc/g/Pa
T	= temperature, K
t	= time, day
T_{air}	= ambient air temperature, K
T_{solar}	= temperature component caused by solar radiation, K
∇^2	= Laplacian operator

Introduction

TO aid in concealment of the firing position of tactical missiles, a class of solid propellants has been developed which produces a relatively low amount of smoke upon burning. Such propellants are commonly referred to as "smokeless." Unfortunately, the ingredients added to the solid propellants to reduce smoke result in an increase of gases produced by chemical reactions in the solid propellant. Under certain environmental conditions these gases can accumulate and produce stresses within the grain sufficiently large to cause cracks or fissures to develop in the grain. Such cracks or fissures are assumed to limit the rocket service life.

In early work, service life predictions for motors with gassing problems were made by subjecting small samples of the propellant to constant temperature environments and noting the times at which cracking occurred for different temperatures.[1] However, such data were shown to be of limited usefulness.[2] Recent research efforts have indicated that more reliable estimates of service life can be made by combining the use of sample cracking data with the solution of the diffusion equation.[3] The method of analysis described here represents a probabilistic approach to the gas generation problem.[4]

Model Description

The gas generation and diffusion behavior in the propellant grain is assumed to be governed by the following equation[4]:

$$D(\nabla^2 \dot{P}) = \dot{P} + Q/S \tag{1}$$

A composite double base propellant designated as type FZO (Ref. 2) was analyzed in the study described here. The material properties D, S, and Q are assumed temperature dependent for the FZO propellant and may be expressed as

$$D = (20)10^{-7}\exp[-1.075(-3.250 + 10^3/T)](cm^2/day) \tag{2}$$

$$S = (15)10^{-8}\exp[+0.761(-3.275 + 10^3/T)](cc/g/Pa) \tag{3}$$

and

$$Q = (2.9)10^{-3}\exp[-4.306(-2.850 + 10^3/T)](cc/g/day) \tag{4}$$

A combined solution method involving a finite-element model with a step-by-step time integration was used to solve Eq. (1) for a hollow cylindrical geometry.

The temperature of the motor is assumed to be uniform and to be expressed in the form

$$T(t) = T_{air}(t) + T_{solar}(t) \tag{5}$$

The ambient air temperature model for Yuma, Arizona developed by Essenwanger and Dudle at the Army Missile Command[2] was used for the $T_{air}(t)$ component in Eq. (5). The solar radiation model for a typical missile located at Yuma, Arizona has been developed by Martin[2] and can be expressed as

$$T_{solar} = \left(0.8 + 0.2\sin\frac{2\pi(d+250)}{365}\right)\left(15. + 20.\sin\frac{2\pi(h+17)}{24}\right) \tag{6}$$

Three motor sizes were examined as described in Table 1. The propellant was assumed the same in all motors.

The critical pressure at which failure is assumed to occur can be related to the maximum stress in a uniaxial constant

Table 1 Motor dimensions

Motor	Case o.d., cm	Case thickness, cm	Grain i.d., cm
Small	12.7	0.152	4.7
Medium	40.6	0.196	12.7
Large	101.6	0.229	40.1

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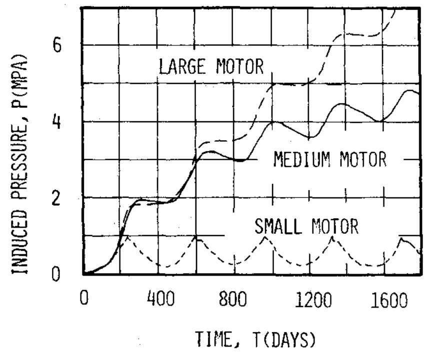

Fig. 1 Influence of motor size on induced pressure.

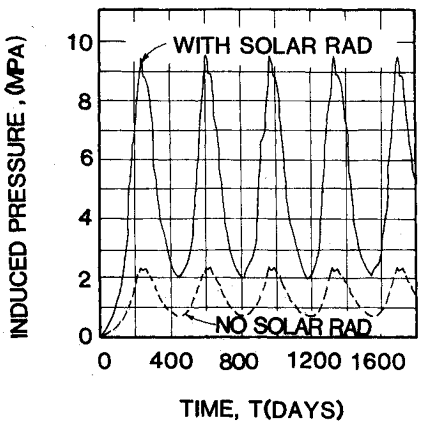

Fig. 2 Influence of solar radiation on induced pressure in small motor.

strain rate test.[2] The critical pressure can be expressed as

$$P_c=(9.936)10^5(t/a_T)^{-0.048}(\mathrm{Pa}) \tag{7}$$

The temperature shift function a_T may be expressed as

$$\log a_T=-8.86(T-30/(71.6+T') \tag{8}$$

where T' is expressed in degrees Celsius.

Induced Pressure Histories

The pressure histories for the three motors described in Table 1 are illustrated in Fig. 1 for a period of approximately five years. Steady-state pressure conditions are seen to be dependent on motor size. The effect of the solar radiation component of temperature $T_{\mathrm{solar}}(t)$ is illustrated in Fig. 2 for the small motor and is seen to be significant.

Probabilistic Service Life Predictions

As can be seen from Fig. 1, the induced pressure does not respond instantaneously to rapid changes in temperature. The response is very slow and can be regarded as deterministic. On the other hand, the critical pressure described in Eq. (7) does fluctuate in an approximate random manner due to the approximate random nature of the temperature fluctuations. Also, variations in the critical pressure occur due to variations in the material composition and test conditions. It seems reasonable to treat the critical pressure as a random variable with an associated distribution function whose mean value fluctuates with temperature.

If the occurrences in a 24-hour period are assumed to constitute a cycle, the probability of failure during this cycle

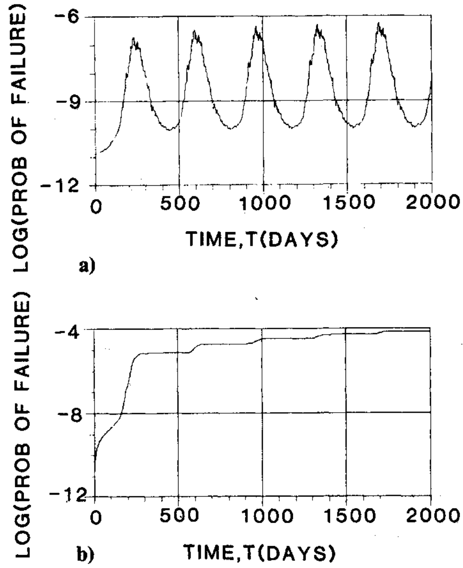

Fig. 3 Probability of failure of small motor. a) daily, b) comulative.

may be computed as

$$(p_f)_i=\int_{P_i}^{\infty} f(x)\,\mathrm{d}x \tag{9}$$

If $(p_f)_i$ is small, the probability of failure for a motor subjected to N cycles is

$$(P_f)_N=\sum_{i=1}^{N}(p_f)_i \tag{10}$$

The service life of the motor is reached at the cycle when the value of $(P_f)_N$ equals the maximum allowable probability of failure for the motor.

Service Life Calculations

The daily probabilities of failure for a period of approximately five years is illustrated for the small motor in Fig. 3a and the cumulative probability of failure in Fig. 3b. These results are based upon an assumed normal distribution function $f(x)$ and a coefficient of variation of 15%. For purposes of illustration, assume an allowable P_f of 1×10^{-5}. Then the service life for the small size motor would be approximately 600 days (1.64 years). Results for other conditions and other size motors can be generated in a similar manner and are described in more detail in Ref. 4.

References

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[2] Martin, D.L. Jr., "The Effect of Stabilizer Depletion and Critical Pressure Ratio on the Service Life Predictions of Smokeless Propellant Motors," U.S. Army Missile Command, Redstone Arsenal, Ala., Report TR-RK-76-3, July 1975.

[3] Cost, T.L. and Weeks, G.E., "Probabilistic Methods in Solid Rocket Motor Structural Integrity Analysis," Athena Engineering Company, Northport, Ala., RK-CR-80-9, Oct. 1980.

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